## Cambridge IGCSE ${ }^{\text {TM }}$



CENTRE NUMBER


CANDIDATE NUMBER


## CAMBRIDGE INTERNATIONAL MATHEMATICS

Paper 6 Investigation and Modelling (Extended)
May/June 2020
1 hour 40 minutes
You must answer on the question paper.
No additional materials are needed.

## INSTRUCTIONS

- Answer both part A (Questions 1 to 6 ) and part B (Questions 7 to 10 ).
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do not use an erasable pen or correction fluid.
- Do not write on any bar codes.
- You should use a graphic display calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly, including sketches, to gain full marks for correct methods.
- In this paper you will be awarded marks for providing full reasons, examples and steps in your working to communicate your mathematics clearly and precisely.


## INFORMATION

- The total mark for this paper is 60 .
- The number of marks for each question or part question is shown in brackets [ ].


## Answer both parts A and B.

## A INVESTIGATION (QUESTIONS 1 to 6)

## DOTTY POLYGONS (30 marks)

## You are advised to spend no more than 50 minutes on this part.

This investigation is about the number of dots in shapes that are regular polygons.
For any polygon

- $\quad p$ is the number of sides
- $n$ is the number of dots on one side
- there are the same number of dots on each side.

1 This is a dotty triangle.


In this dotty triangle, $p=3$ and $n=4$.
(a) Look at the numbers of dots in each row of the triangle.

Complete this sum for the total number of dots in the triangle.

$$
\begin{equation*}
1+2+3+ \tag{1}
\end{equation*}
$$

$\qquad$ $=$
(b) Show that $\frac{n^{2}}{2}+\frac{n}{2}$ gives the correct number of dots when $n=10$.

2 The diagram shows the first four dotty triangles. The number of dots added each time is $d$.
-••


$n=1$
$n=2$
$n=3$
$n=4$
$d=1$
$d=2$
$d=3$
$d=4$

So, for dotty triangles, $d=n$.
This diagram shows the first four dotty squares.

$$
n=1
$$

$$
n=2
$$

$$
n=3
$$

$$
n=4
$$

$$
d=1
$$

$$
d=3
$$

$$
d=5
$$

$$
d=7
$$

(a) Find an expression, in terms of $n$, for the total number of dots in the $n$th dotty square.
(b) For dotty squares, find a formula for $d$ in terms of $n$.

3 (a) A formula for $d$, in terms of $p$ (the number of sides) and $n$ is

$$
d=(p-2) n-p+3 .
$$

For dotty pentagons, show that the formula becomes $d=3 n-2$.
(b) This diagram shows the first three dotty pentagons.


Dotty pentagons grow along the grey lines.
This diagram shows how to form the first three dotty pentagons.


Complete the diagram to show the 4th and 5th dotty pentagons.

4 (a) This table shows the total number of dots in some dotty polygons.
Use Question 2, Question 3 and any patterns you notice to help you complete this table.

|  |  | Position of dotty polygon in its sequence |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Polygon | $p$ | 1 st | 2nd | 3 rd | 4th | 5 th | $n$th |  |
| Triangle | 3 | 1 | 3 | 6 | 10 |  |  | $\frac{n^{2}}{2}+\frac{n}{2}$ |
| Square | 4 | 1 | 4 | 9 | 16 |  |  |  |
| Pentagon | 5 | 1 | 5 | 12 |  |  |  |  |
| Hexagon | 6 | 1 | 6 |  |  |  |  |  |

(b) Complete this expression, in terms of $n$, for the total number of dots in any dotty pentagon.

5 (a) Use Question 4 and any patterns you notice to help you complete this table.

| Polygon | $p$ | Total number of dots <br> $(n)$ |
| :--- | :---: | :---: |
| Triangle | 3 | $\frac{n^{2}}{2}+\frac{n}{2}$ |
| Square | 4 |  |
| Pentagon | 5 |  |
| Hexagon | 6 |  |

(b) Find, in terms of $n$ and $p$, an expression for the total number of dots in any dotty polygon.

6 When $p=50$,
$\begin{aligned} & \text { The total number of dots in } \\ & \text { the } n \text {th dotty polygon }+865\end{aligned}=\quad \begin{gathered}\text { The total number of dots in } \\ \text { the }(n+1) \text { th dotty polygon. }\end{gathered}$

Find the value of $n$.

## B MODELLING (QUESTIONS 7 to 10)

## STOPPING A CAR (30 marks)

You are advised to spend no more than 50 minutes on this part.

This task is about the distance a car travels as it stops.
7 (a) Show that a speed of $130 \mathrm{~km} / \mathrm{h}$ is approximately $36 \mathrm{~m} / \mathrm{s}$.
(b) A car travels at $130 \mathrm{~km} / \mathrm{h}$.

A model for the distance travelled, $d$ metres, in time $t$ seconds, is $d=36 t$.
Write a similar model for a car travelling at $80 \mathrm{~km} / \mathrm{h}$.
(c) On the diagram, sketch both models for $0 \leqslant t \leqslant 3$.

(d) On your sketch, shade the region showing the distances travelled at speeds from $80 \mathrm{~km} / \mathrm{h}$ to $130 \mathrm{~km} / \mathrm{h}$ for $0 \leqslant t \leqslant 3$.

8 When a driver looks at a mobile phone they do not look at the road.
On average, they look at their mobile phone for 2 seconds.
For speeds between $80 \mathrm{~km} / \mathrm{h}$ and $130 \mathrm{~km} / \mathrm{h}$, find the range of distances that the car travels in these 2 seconds.
$\qquad$ $\leqslant$ distance $\leqslant$

9 A car continues to travel after the brakes are applied.
The distance travelled is the braking distance, $b$ metres.
A model for $b$ is $b=\frac{v^{2}}{20 f}$
where $\quad v$ is the speed of the car in metres per second $f$ is a measure of grip the tyre has on the road.
(a) When the road is dry the value of $f$ is 0.7 .

Write, and simplify, the model for $b$ when the road is dry.
(b) When the road is icy the value of $f$ is 0.3 .

Write, and simplify, the model for $b$ when the road is icy.
(c) On the diagram, sketch both models for $0 \leqslant v \leqslant 20$.

(d) A car travels at $60 \mathrm{~km} / \mathrm{h}$.
(i) On the diagram in part (c), sketch a vertical line at $60 \mathrm{~km} / \mathrm{h}$.
(ii) Find how much greater the braking distance is when the road is icy than when the road is dry.

Question 10 is printed on the next page.

10 Glen is driving his car when an emergency happens.
He usually takes 0.7 seconds to react before braking.
Because he is looking at his mobile phone he takes an extra 2 seconds to react.
The weather is wet and the measure of grip the tyre has on the road is 0.5 . Glen is driving at $x \mathrm{~km} / \mathrm{h}$.
(a) Show that $x \mathrm{~km} / \mathrm{h}$ is approximately $0.28 x \mathrm{~m} / \mathrm{s}$.
(b) The total stopping distance is the distance the car travels from when the emergency happens to when the car stops.

Use Question 8 and Question 9 to find, in terms of $x$, a model for Glen's total stopping distance in metres.
Give your answer as simply as possible.

